Journée Régionale des Doctorants en Automatique, 24 juin 2025

Homogeneous predictor feedback for a 1D reaction diffusion equation with input delay

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Delay compensation in parabolic partial differential equations (PDEs) has become a critical topic, as control actions in complex systems arising in fields such as biology, chemistry, and spatial ecology, can be significantly delayed. The presence of delays increases mathematical complexity, necessitating specialized techniques for stability analysis, numerical implementation, and control design. Control design for complex systems modeled by PDEs becomes even more challenging with input delays. The boundary stabilization of the one-dimensional reaction-diffusion equation with input delay was first introduced and tackled using the backstepping method. An alternative method for stabilizing parabolic PDEs is the modal decomposition approach, which separates a finite-dimensional unstable component from a stable infinite-dimensional part of the PDE and subsequently designs a controller based on the unstable modes. Following this strategy and utilizing Artstein's transformation, a linear predictor feedback based on the unstable modes was developed for the 1D reaction-diffusion equation. However, linear control has notable drawbacks, including the peaking effect and significant overshoot. A homogeneous controller can mitigate these issues, achieving rapid convergence without peaking and with minimal overshoot.

In this presentation, we introduce a nonlinear boundary stabilization of a 1D reaction-diffusion equation with input delay. Using the modal decomposition approach, we propose a homogeneous-based predictor feedback for stabilizing the unstable modes. We prove the stability of the closed-loop system via the construction of a suitable Lyapunov functional. We present numerical simulations to support the analytical results and compare our proposed controller to linear predictor feedback regarding closed-loop performance and peaking effect.

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